GJ spent 51 hours sleeping the first week of the quarter. Each week afterwards, GJ's sleep time was 3% less than the previous week. If the quarter was 11 weeks long, how much time did GJ sleep over the entire quarter ?

SCORE: _____ / 10 PTS

$51 + 51(0.97) + 51(0.97)^2 + \cdots + 51(0.97)^{10}$ GEOMETRIC, $r = 0.97$

$= \dfrac{51(1-0.97^{11})}{1-0.97} = 483.99$ HOURS

Find the sum of the series $232 + 225 + 218 + 211 + 204 + \cdots - 377$.

SCORE: _____ / 15 PTS

ARITHMETIC, $d = -7$

$232 - 7(n-1) = -377$

$-7(n-1) = -609$

$n - 1 = 87$

$n = 88$

$S_{88} = \frac{1}{2}(88)(232 + -377) = -6380$

HJ is standing 20 feet from IJ, who is 4 feet tall. HJ throws a football at 25 feet per second in IJ's direction, at an angle of 61.93° with the horizontal, from an initial height of 6 feet.

SCORE: _____ / 25 PTS

NOTE: $\sin 61.93^\circ = \frac{15}{17}$ **and** $\cos 61.93^\circ = \frac{8}{17}$

[a] Write parametric equations for the position of the football.

$x = (25 \cos 61.93^\circ)t = \frac{200}{17}t$

$y = 6 + (25 \sin 61.93^\circ)t - 16t^2 = 6 + \frac{375}{17}t - 16t^2$

[b] Does the football hit IJ, go over IJ's head, or hit the ground before reaching IJ ?

$x = 20$

$\frac{200}{17}t = 20$

$t = 1.7$

$y = 6 + \frac{375}{17}(1.7) - 16(1.7)^2$

$y = -2.74 < 0$

THE FOOTBALL HITS THE GROUND BEFORE REACHING IJ

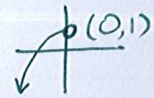
Describe the difference between the curves with parametric equations $\begin{array}{l} x=-e^t \\ y=1-e^{2t} \end{array}$ and $\begin{array}{l} x=\sin t \\ y=\cos^2 t \end{array}$. **SCORE: _____ / 10 PTS**

Discuss the rectangular equation(s) of the graphs, as well as the orientation and portion of the graph corresponding to the parametric equations.

BOTH CURVES CORRESPOND TO $y=1-x^2$ 

AS t GOES FROM $-\infty$ TO ∞

$x=-e^t$ GOES FROM ≈ 0 TO $-\infty$ BUT $x=\sin t$ OSCILLATES BETWEEN -1 AND



Use sigma notation to write the series $\overset{3!}{\dfrac{6}{128}} \xrightarrow{\times 4} - \overset{4!}{\dfrac{24}{192}} \xrightarrow{\times 5} + \overset{5!}{\dfrac{120}{288}} \xrightarrow{\times 6} - \overset{6!}{\dfrac{720}{432}} + \cdots - \overset{8!}{\dfrac{40320}{972}}$. $\leftarrow$ GEOMETRIC $r=-1.5$ **SCORE: _____ / 15 PTS**

$$\sum_{n=1}^{6} \frac{(n+2)!}{32(-1.5)^{n-1}} \quad \text{OR} \quad \sum_{n=3}^{8} \frac{n!}{32(-1.5)^{n-3}} \quad \text{OR} \quad \sum_{n=0}^{5} \frac{(n+3)!}{32(-1.5)^{n}}$$

Using mathematical induction, prove that $\displaystyle\sum_{i=1}^{n} 3^{n+1} = \frac{3^{n+2}-9}{2}$ for all positive integers n. **SCORE: _____ / 25 PTS**

Do NOT use the finite geometric series formula in your proof.

BASIS STEP: WHEN $n=1$, $\displaystyle\sum_{i=1}^{1} 3^{i+1} = 3^2 = 9$

$$\frac{3^3-9}{2} = \frac{18}{2} = 9$$

INDUCTIVE STEP:

ASSUME $\displaystyle\sum_{i=1}^{k} 3^{i+1} = \frac{3^{k+2}-9}{2}$ FOR SOME ARBITRARY BUT PARTICULAR INTEGER k

$$\sum_{i=1}^{k+1} 3^{i+1} = \sum_{i=1}^{k} 3^{i+1} + 3^{k+2}$$

$$= \frac{3^{k+2}-9}{2} + 3^{k+2}$$

$$= \frac{3^{k+2}-9+2\cdot 3^{k+2}}{2}$$

$$= \frac{3\cdot 3^{k+2}-9}{2}$$

$$= \frac{3^{k+3}-9}{2}$$

$$= \frac{3^{(k+1)+2}-9}{2}$$

BY MI,

$$\sum_{i=1}^{n} 3^{i+1} = \frac{3^{n+2}-9}{2}$$

FOR ALL POSITIVE INTEGERS n

Simplify $\dfrac{(8n-2)!}{(8n+1)!}$.

SCORE: _____ / 10 PTS

$$\frac{\cancel{(8n-2)!}}{(8n+1)8n(8n-1)\cancel{(8n-2)!}} = \frac{1}{8n(8n+1)(8n-1)}$$

Eliminate the parameter to find rectangular equations corresponding to the parametric equations $\begin{array}{l} x = \dfrac{t+1}{t-3} \\ y = \dfrac{t}{2-t} \end{array}$.

SCORE: _____ / 15 PTS

For your final answer, write y as a simplified function of x.

$$x(t-3) = t+1$$
$$xt - 3x = t+1$$
$$xt - t = 3x+1$$
$$(x-1)t = 3x+1$$
$$t = \frac{3x+1}{x-1}$$

$$y = \frac{\frac{3x+1}{x-1}}{2 - \frac{3x+1}{x-1}}$$
$$= \frac{3x+1}{2(x-1)-(3x+1)}$$
$$= \frac{3x+1}{-x-3} = -\frac{3x+1}{x+3}$$

Consider the expression $(19x^7 - 13x^4)^{21}$.

SCORE: _____ / 25 PTS

[a] Write the expansion of the expression using sigma notation. **Your answer may use ! but not $_nC_r$ notation.** Simplify all exponents.

$$\sum_{r=0}^{21} {}_{21}C_r (19x^7)^{21-r}(-13x^4)^r = \sum_{r=0}^{21} \frac{21!}{r!(21-r)!} 19^{21-r}(-13)^r x^{7(21-r)+4r}$$
$$= \sum_{r=0}^{21} \frac{21!}{r!(21-r)!} 19^{21-r}(-13)^r x^{147-3r}$$

[b] Find the coefficient of x^{108} in the expansion. **Your answer may use ! but not $_nC_r$ notation.**

$$147-3r = 108$$
$$-3r = -39$$
$$r = 13$$

$$\frac{21!}{13!8!} 19^8(-13)^{13} = -\frac{21!}{13!8!} 19^8 13^{13}$$